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ROOM ACOUSTICAL SIMULATION ALGORITHM BASED ON THE FREE PATH DISTRIBUTION

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A new algorithm is presented which provides estimates of impulse responses in rooms. It is applicable to arbitrary shaped rooms, thus including non-diffuse spaces like workrooms or offices. In the latter cases, for instance, sound propagation curves are of interest to be applied in noise control. In the case of concert halls and opera houses, the method enables very fast predictions of room acoustical criteria like reverberation time, strength or clarity. The method is based on a low-resolved ray tracing and recording of the free paths. Estimates of impulse responses are derived from evaluation of the free path distribution and of the free path transition probabilities.

1. INTRODUCTION

Geometrical acoustics is by far the most successful technique for prediction of sound field parameters in large rooms. Although it is not an exact field theory, it provides reasonable estimates of sound fields in enclosures. Investigations of the history of sound rays travelling in rooms were the basis for statistical reverberation theory and thus for Sabine's and Eyring's equations. Sound propagation is usually considered as a reflection sequence with energy loss of $(1-\alpha)$ per reflection and a global energy loss of

$$w(v) = w_0(1-\alpha)^v = w_0 \exp(v \ln(1-\bar{\alpha})) \tag{1}$$

with v denoting the number of reflections and $\bar{\alpha}$ the mean absorption coefficient. Usually, v is set proportional to the mean reflection rate $\bar{n}$ according to $v = \bar{n}t = ct/\bar{l}$, assuming the free paths $\bar{l} = c/\bar{n}$ equally long. The decay constant of the exponential function in a room with volume V and surface S is thus $\bar{n}\ln(1-\bar{\alpha})$ with the well-known $\bar{n} = cS/4V$ in the "diffuse field" [1]. The validity of diffuse field conditions has been discussed very intensively in the literature. After all, it can be stated that simple diffuse field theory never applies exactly, but gives reasonable estimates in by far the most cases. From the expectation value of the energy density impulse response

$$w(t) = w_0 \exp(\bar{n}t \ln(1-\bar{\alpha})), \tag{2}$$

one can derive the steady state energy density (P = sound power, $A = -S \ln(1 - \bar{\alpha})$)

$$w_{diffuse} = 4P/cA \tag{3}$$

and the reverberation time T in s (with V in m^3 and A in m^2)

$$T = 0{\cdot}16V/A. \tag{4}$$

If, however, the free paths are quite different as, for instance, in an extremely flat room, it is no news that equation (1) and hence, Eyring's expectation value for the reverberation time is not correct (see Figure 1).

This problem was investigated by Kuttruff [2], who derived a correction term for Eyring's equation, which depends on the variance of the free path distribution and hence—at least partly—on the room shape. Barron and Lee [3] derived estimates for room acoustical criteria from exponential decay curves based on the reverberation time T. Adding direct sound energy makes receiver-dependent results possible. Clarity and strength, for instance, can be calculated rather easily by using T and the room volume (usually called "Barron's revised theory"). This approach is enhanced further by improving the interpretation of the exponential function in order to obtain a better estimate of the relation between sound power and the total

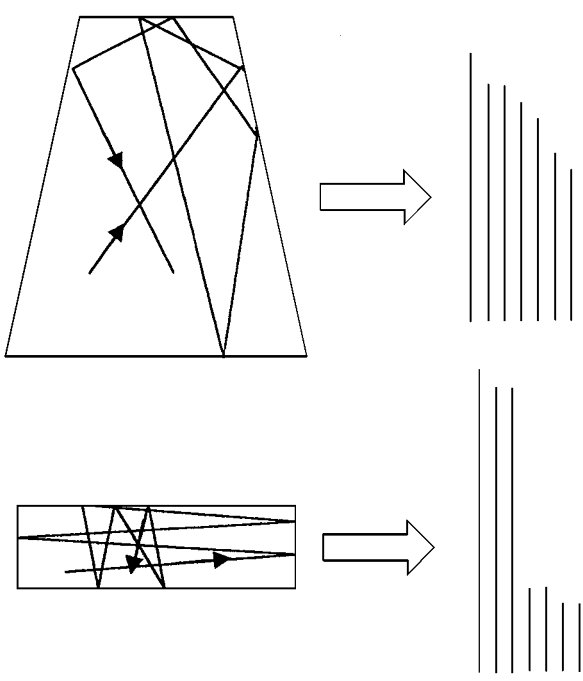

Figure 1. Relation between room shape and free paths, top: "normally" shaped room, bottom: flat room. The lengths of free paths are recorded and put into an order according to their length.

sound energy in the room [4],

$$w_{diffuse} = (4P/cA)\mathrm{e}^{-A/S}, \tag{5}$$

which might be used also in combination with the revised theory described by Barron and Lee. The improvements of the statistical reverberation theory based on mean values and variances, however, cannot be expected to give much more information than expressed in equation (1)–(5).

On the other hand, advanced room acoustical computer simulations can be used to create room impulse responses at any point in a room quite accurately, provided geometrical acoustics apply (sufficient at not too low frequencies). The drawback of the latter method is that substantial calculation is required for generating impulse responses.

The approach reported here is based on the idea that the room shape and the wall scattering are related with a specific free path distribution [5]. The distribution, however, is accounted for with its full statistical content without aiming at mean values of variances, and it is obvious that the free path distribution and corresponding transition probabilities can very easily be found by ray tracing. The number of rays necessary to obtain the data is very small compared to that for the conventional algorithm.

2. THEORY

The free path distribution and the corresponding free path transition distribution of sound rays can easily be determined by ray tracing. The distribution can be expressed by means of a matrix which contains the free paths and the absorption coefficients involved in the reflections (see Figure 2).

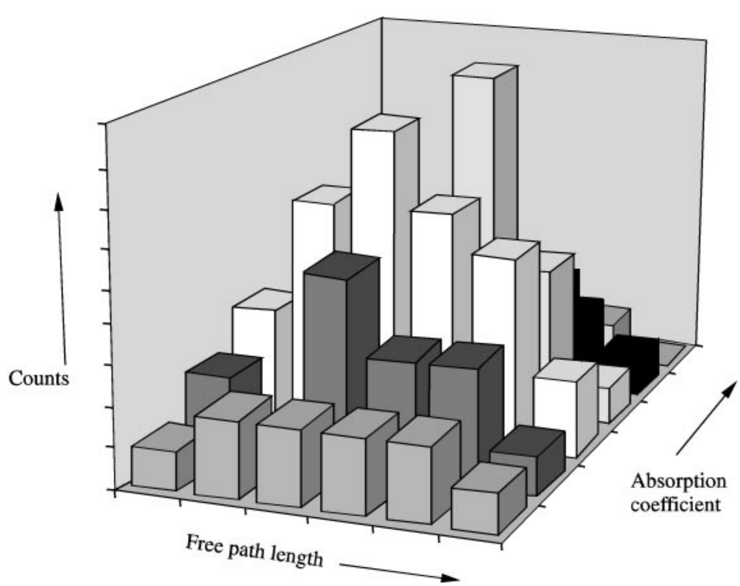

Figure 2. Matrix of statistical distribution of free path lengths and absorption coefficients.

Imagine a flat room with different path lengths and absorption of reflections between floor and ceiling and between walls (see Figure 1, bottom). The decay process can be divided into sub-processes of decays with each having individual decay constants $n_i \ln(1-\alpha_j)$ with n_i and α_j denoting a category of reflection rates (free paths) and absorption coefficients respectively. Each category contributes to the global energy in the room. It is related with an exponential decay component w_{ij}

$$w_{ij}(t) = w_{ij,0} \exp(n_i t \ln(1-\alpha_j)) \tag{6}$$

and with a total energy density proportional to

$$w'_{global}(t) = \sum_{i,j} w_{ij}(t). \tag{7}$$

It is clear that ergodicity is a basic assumption because the time history of rays is estimated from the global spatial distribution of free paths.

3. TRANSITION PROBABILITY

Now, a further problem must be solved because the sub-processes ij are not closed. A certain energy transition between the processes occurs. In the case of specular reflections between parallel walls the transition probability might be small since the reflection paths stay within their group for a long time. If, however, diffuse scattering is involved, energy may jump from one category into another with a rather high probability. To achieve the possibility of accounting for the transition, the matrix scheme is extended by another dimension: the free paths before and after each reflection in columns and rows respectively (see below, the example in Figure 5). The absorption coefficients can be addressed to other dimensions. The vector obtained after summation over columns (or rows) forms the general free path distribution, as given in reference [2].

The modified formalism of construction of impulse responses is illustrated in Figure 3. The energy category $w_{ij}(t)$ is generally following the law described by equation (6), but it suffers from energy loss according to the transition matrix elements corresponding to $ij \to i'j'$, and, *vice versa*, it receives energy according to the sum over the row of the transition matrix $\Sigma i'j' \to ij$. In a first approach, the energy can be transferred in discrete steps according to the individual free path delays (see Figure 3). Several other variants may be investigated in future.

Finally, the total energy density in J/m^3 is obtained after adding the receiver-dependent direct sound to the "room sound field" (r is the distance source-receiver, P the sound power and C the constant for normalization of direct and reflected sound):

$$w_{total}(t) = \left.\frac{P}{4\pi c r^2}\right|_{t=r/c} + \left. C w'_{global}(t)\right|_{t>r/c+1/\bar{n}}. \tag{8}$$

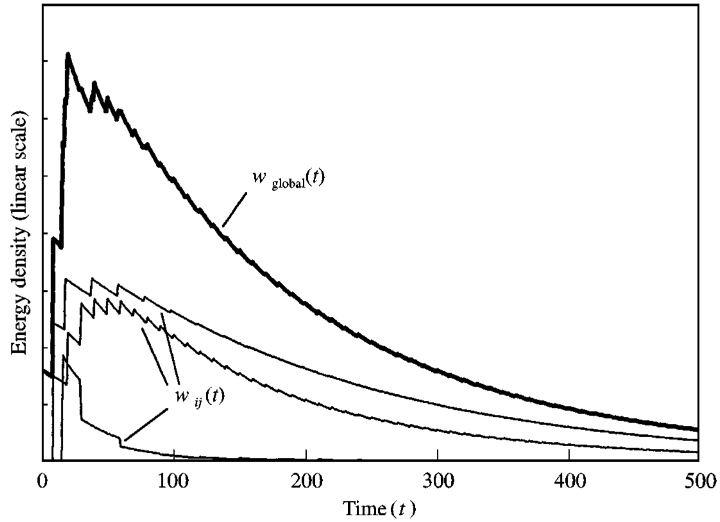

Figure 3. Individual and global decay processes with transition between free path and absorption categories.

The normalization constant C depends on the choice of the specific ray-tracing method. In the case of global tracing (without specific receivers, N is the number of rays used, t_{max} the upper temporal limit of tracing each ray),

$$C_{total} = 4P/Nct_{max}S. \tag{9}$$

In the case of receiver-dependent ray counting (r_d the detector radius), C according to equation (9) is to be multiplied by the ratio of the room volume and the receiver volume $3r_d^3 V/4\pi$.

4. EXAMPLES

To check the plausibility of the algorithm, an extremely flat room serves as an example. Figure 4 shows decay curves for specular and diffuse reflections at floor and ceiling. For comparison, the linear level decay according to Eyring's formula is added. The results agree well with the results from conventional ray tracing. The calculation time for post-processing was below 1 s, while the matrices were determined with 3600 rays, which lasted 2 min.

The second example involves a coupled room. The absorption coefficients are uniformly $\alpha = 0{\cdot}15$. The larger room has a volume $V_1 = 1000\ \mathrm{m}^3 = 10 \times 10 \times 10\ \mathrm{m}^3$, the smaller has $\frac{1}{8}$ the size with $V_2 = 125\ \mathrm{m}^3 = 5 \times 5 \times 5\ \mathrm{m}^3$. The opening between the rooms is a door of 1 m width and 5 m height. The sound source is placed in a corner of the small room. At first, the distribution of free path is discussed (see Figure 5). Here, the matrix has just the two dimensions of the free paths since the absorption is uniformly distributed. Due to the fact that the source is located in the small room, the partial sound field up to path lengths of 5 m and the corresponding

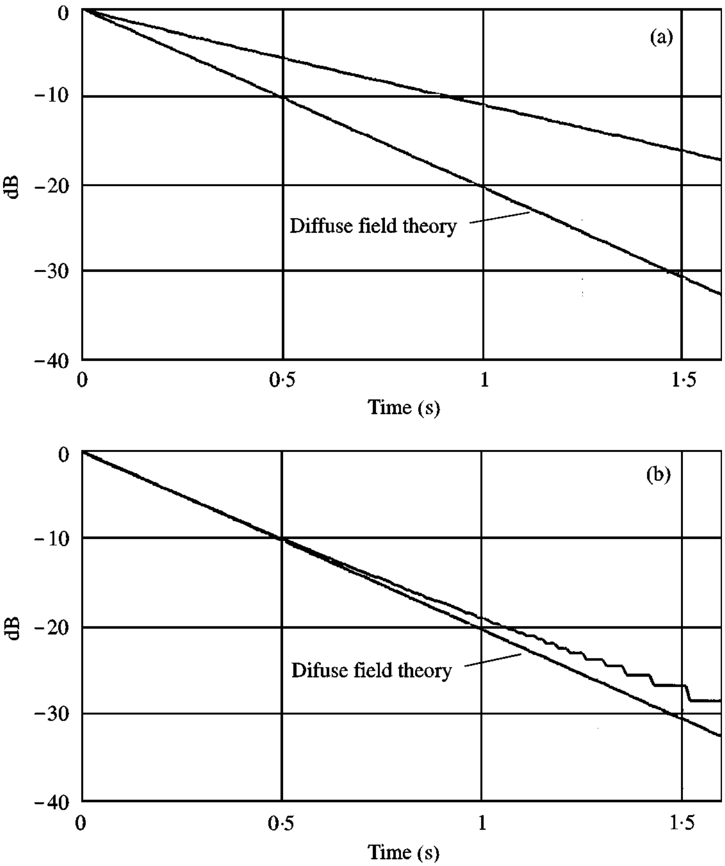

Figure 4. Sound propagation curves in a flat room ($100 \times 100 \times 4\ m^3$): (a) specular; (b) diffuse reflections.

distribution looks very similar to the expectation value of the small room alone. Projections on one of axes of the distribution between 0 and 5 m path lengths agree very well with the free path distribution of cubic rooms as illustrated by Kuttruff [2]. Apart from that, one can see that some rays are entering the large room with in principle the same path length behaviour; hence a flat distribution between 0 and 8 m and a peak around 10 m path length is obtained.

Finally, Figure 6(b) shows the global energy impulse responses composed according to equation (7). For comparison, in Figure 6(a) the transition effect is neglected by disabling the transition in the energetic summation. Including the transition effects the decay slopes are estimated almost equally. The intersection of the two level lines, however, is found to be some decibels lower if transition of energy is allowed to occur.

5. CONCLUSION

The method of estimating room impulse responses from free path distributions offers promising new insight into the statistical features in geometrical room

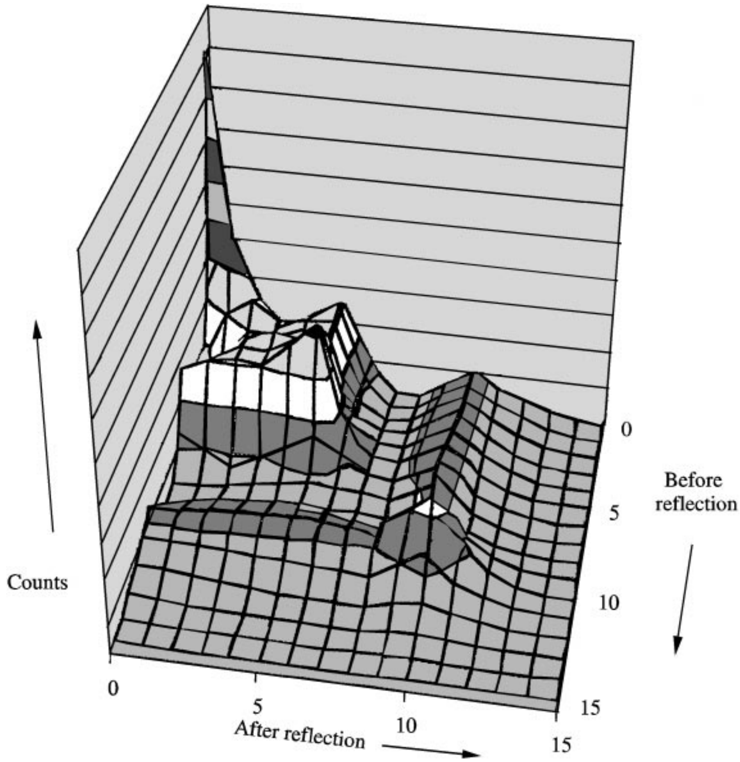

Figure 5. Free path distribution matrix of coupled rooms with uniform absorption.

acoustics. Since the distribution can very easily be recorded by ray tracing, it is worthwhile to investigate this or similar algorithms in near future. Although not yet tested, it should be possible to construct receiver-dependent impulse responses similar to those of Barron's revised theory and, accordingly, room acoustical single number quantities. In the case of noise control in rooms, the parameter "strength" or other normalized levels can be predicted for determination of sound propagation curves. This is particularly interesting since the method allows extremely non-diffuse spaces to be simulated very quickly. The statistical mean values of free paths and absorption coefficients are not needed.

It should be noted, however, that this approach is still a statistical method and cannot yield exact decay curves. It is intended for an intermediate level of complexity and accuracy in order to close a gap between complete ray tracing or image source processes and simple formulae. Therefore, it is quite useful for the so-called "electroacoustic simulations" of public address systems in rooms. Today, the programs have very nice user interfaces and graphical outputs, but they suffer from the preposition of the diffuse field conditions in the late decay.

Further investigations should be focussed on the limitations of the model due to higher order statistical coupling between the categories, the validity of the spatial independence of the paths and the temporal development of the spatial energy density. Further work must be done to adapt this model into room acoustical

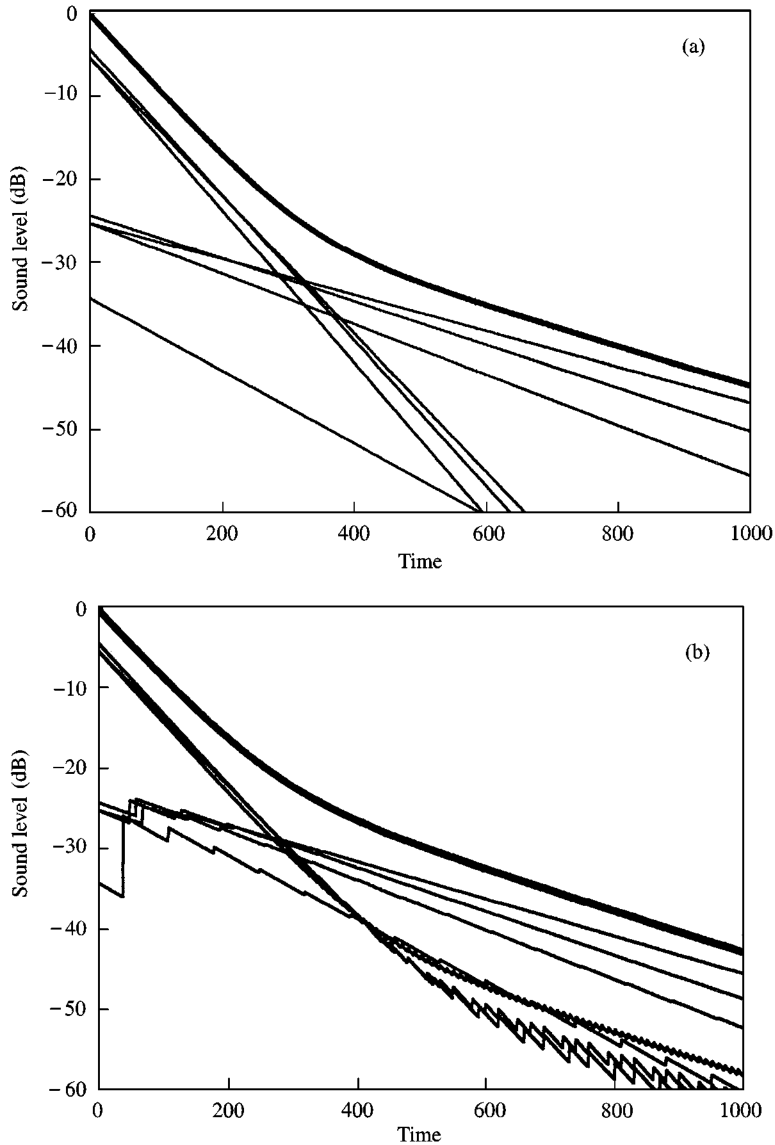

Figure 6. Energy decay curves of a coupled room. Examples of individual curves of free path categories and total decay curve (——). (a) Without transition between the categories; (b) with transition between the categories.

simulation software for fast hybrid calculation of early and late parts of impulse responses.

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